Calhoun: The NPS Institutional Archive

DSpace Repository

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

1960

Further developments in the theory of convective currents

Chase, Edgar M.

http://hdl.handle.net/10945/12242

Downloaded from NPS Archive: Calhoun

DUDLEY KNOX LIBRARY

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

NPS ARCHIVE
1960
CHASE, E.

FURTHER DEVELOPMENTS IN THE THEORY OF CONVECTIVE CURRENTS

EDGAR M. CHASE

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY CA 9[illegible]-5101

Library
U. S. Naval Postgraduate School
Monterey, California

FURTHER DEVELOPMENTS IN THE THEORY OF CONVECTIVE CURRENTS

* * * * *

Edgar M. Chase

FURTHER DEVELOPMENTS
IN THE THEORY OF
CONVECTIVE CURRENTS

by

Edgar M. Chase

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

1960

NPS Archive
1960
Chase, E

Library
U. S. Naval Postgraduate School
Monterey, California

FURTHER DEVELOPMENTS
IN THE THEORY OF
CONVECTIVE CURRENTS

by

Edgar M. Chase

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
IN
METEOROLOGY
from the
United States Naval Postgraduate School

ABSTRACT

The Haltiner theory of saturated steady state convective currents which includes the transfer of heat and momentum by lateral diffusion as well as the entrainment of environmental air is discussed. A modified model is presented which includes, as a separate term, the drag of the liquid water carried along by the ascending current. The system of equations is integrated numerically for several cases previously considered by Haltiner allowing entrainment to assume negative values (detrainment) and the results are compared. The results are in good agreement, and show that the net effect of the opposing actions of liquid water drag and detrainment is a reduction in the computed vertical velocities. This reduction appears to be a correction in the right direction on the basis of a comparison to some observed data.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor G.J. Haltiner of the U.S. Naval Postgraduate school in this investigation.

TABLE OF CONTENTS

Section	Title	Page
1.	Introduction	1
2.	The System of Equations	3
3.	The Complete Model	11
4.	Computations and Results	13
5.	Conclusions	16
6.	Bibliography	25

LIST OF ILLUSTRATIONS

Figure Page

1. Vertical velocity w, fractional mass increase M/M_o, liquid water content ℓ and temperature excess over the environment ΔT as a function of height for Case (a) 18

2. Vertical velocity w, fractional mass increase M/M_o, liquid water content ℓ and temperature excess over the environment ΔT as a function of height for Case (b) 19

3. Temperature excess over environment ΔT as a function of height for Case (a), Case (b) and after Haltiner 20

4. Fractional mass increase M/M_o as a function of height for Case (a), Case (b), and after Haltiner 21

5. Vertical velocity w as a function of height for Case (a), Case (b), and after Haltiner 22

6. Liquid water content ℓ as a function of height for Case (a), Case (b), and after Haltiner 23

Table

1. Comparison of maximum values of ΔT, M/M_o, w, ℓ and cloud height, h, with maximum values after Haltiner 24

1. Introduction

The concept of simple "adiabatic parcel ascent" as a first approximation to convective phenomena has long been in use; it has, however, a number of shortcomings. As a result of evidence (Stommel,[8]) that a rising convective current acts like a jet, dragging in surrounding air as it progresses upward, Austin and Fleisher suggested that this process, called "dynamic entrainment", is necessary to satisfy mass continuity. [1] Haltiner[4] summarized the work of Houghton and Cramer,[5] Bunker,[2] and Malkus,[6] and suggested a theory of saturated convective currents which includes the effects of lateral diffusion of heat and momentum, and extended computations to elevations typical of large cumulus clouds.

Haltiner[4] derived a system of equations which describe a saturated convective current. The derivation of these equations is essentially the same as the derivation of the modified system of equations presented below. The Haltiner model assumes that during detrainment the cross-sectional area of the rising column remains constant. However, when mass continuity considerations calls for the ejection of mass (detrainment), a second system of equations is applied based on the assumptions that the rate of entrainment remains identically zero and mass continuity is

satisfied by an increasing cross-sectional area in the rising air. In this paper the detrainment phase will be treated by retaining the assumption that the cross-sectional area of the rising air remains constant, thus permitting the entrainment to assume negative values.

The effect of the drag of the liquid water carried along in the rising air was not explicitly considered in the Haltiner model, although it can be qualitatively included by adjusting the rate of diffusion of momentum. Here the drag effect of the liquid water will be specifically included as a separate term.

The following notation will be used throughout this paper:

T	temperature
z	height
q	specific humidity
c_p	specific heat of air at constant pressure
L	latent heat of vaporization
g	gravity
w	velocity
k	diffusion coefficient
R_d	gas constant for dry air
R_v	gas constant for water vapor
N	$\ln M$
M	Mass rate of flow per unit time

χ	"specific humidity" of liquid water
σ	cross-sectional area of rising column
ρ	density
p	pressure
γ_e	lapse rate

subscripts:

e	environment
v	virtual
o	initial value

2. The system of equations

a. The continuity equation.

Consider a vertical current of air in which the velocity, w, is horizontally uniform. If the field of vertical velocity is not uniform w may be considered to be the mean value in the horizontal at any given level. The mass of air per unit time flowing past any given level, z, is given by

$$M = \rho w \sigma$$

where M is the mass, ρ the density and σ the cross-sectional area of the vertical current. Under steady state conditions the mass at an adjacent level $z + dz$ is the sum of the mass rate of flow at level z, and air entrainment from z to $z + dz$. Thus,

$$(1) \qquad \frac{dM}{M} = \frac{d\rho}{\rho} + \frac{dw}{w} + \frac{d\sigma}{\sigma}$$

Defining a parameter $N = \ln M$, we may write

$$(2) \quad \frac{dN}{dz} = \frac{1}{\rho}\frac{d\rho}{dz} + \frac{1}{w}\frac{dw}{dz} + \frac{1}{\sigma}\frac{d\sigma}{dz}$$

Differentiation of the equation of state $p = R_d \rho T_v$ yields

$$(3) \quad \frac{1}{\rho}\frac{d\rho}{dz} = \frac{1}{p}\frac{dp}{dz} - \frac{1}{T_v}\frac{dT_v}{dz}$$

We next make the assumption that at any level the pressure in the rising air is the same as that of the environment, then $\frac{1}{p}\frac{dp}{dz} = \frac{1}{p_e}\frac{\partial p_e}{\partial z}$

From the hydrostatic equation $\partial p_e / \partial z = -g\rho_e$ it follows that

$$(4) \quad \frac{1}{p}\frac{dp}{dz} = \frac{1}{p_e}\frac{\partial p_e}{\partial z} = -\frac{g}{R_d T_{ve}}$$

We may now eliminate the density term from (2) by substitution from (3) and (4) to yield the continuity equation for the rising air

$$(5) \quad \frac{dN}{dz} = \frac{1}{w}\frac{dw}{dz} + \frac{1}{\sigma}\frac{d\sigma}{dz} - \frac{g}{R_d T_{ve}} - \frac{1}{T_v}\frac{dT_v}{dz}$$

b. The Equation of Motion.

The changing mass of the rising air due to entrainment or detrainment requires the use of an equation of motion for a variable mass. If a mass M is moving at velocity w at time t combines with a second mass dM moving at velocity w_e during a time interval dt

and w + dw represents the velocity of the combined masses at time t + dt, then the momentums at time t and t + dt are related by the equation

$$(M + dM)(w + dw) = Mw + EM\,dt + w_e\,dM$$

Here E represents the external forces per unit mass. This relationship yields the equation of motion

$$(6) \quad \frac{dw}{dt} + \frac{w}{M}\frac{dM}{dt} = E + w_e \frac{1}{M}\frac{dM}{dt}$$

The external forces acting on the system include the pressure and gravity forces, which may be expressed as $g(T_v - T_{ve})/T_{ve}$, and frictional forces F, the latter including the drag of the liquid water carried by the ascending column of air. The equation of motion for the liquid water rising with the air may be written

$$(7) \quad \ell \frac{dw}{dt} = -\ell g + D$$

where ℓ is the mass of liquid water per unit mass of air which we will term the "specific humidity" of the water and D represents the drag of the air acting on the liquid water of mass ℓ. The negative of D is the drag of the liquid water on a unit mass of air. From eq (7), we have

$$-D = -\ell g - \ell \frac{dw}{dt}$$

substituting in eq (6) we obtain

$$\frac{dw}{dt} = -\ell\frac{dw}{dt} - \ell g + g\left(\frac{T_v - T_{ve}}{T_{ve}}\right) - \frac{(w - w_e)}{M}\frac{dM}{dt} + F$$

simplifying we have

$$(1+\ell)\frac{dw}{dt} = g\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right) - (w - w_e)\frac{dN}{dt} + F$$

assuming steady state conditions we obtain

$$(8) \quad w\frac{dw}{dz} = \frac{1}{(1+\ell)}\left[g\left(\frac{T_v - T_{ve}}{T_v} - \ell \right) - w(w - w_e)\frac{dN}{dz} + F \right]$$

c. Lateral diffusion of heat and momentum

In addition to the exchange due to entrainment, turbulent exchange of heat and momentum must also be considered. Priestley [7] (1956) investigated the mechanisms of diffusive heat and momentum exchange in a plume of hot gases. Haltiner [4] suggested that, except for the scale of the phenomena, the diffusive exchange processes in a convective current are similar to the processes defined by Priestley. The effects of lateral diffusion were included in the Haltiner model by the extention of the Priestley equations to convective currents. The heat loss per unit time is taken to be

$$(9) \quad -k_1 c_p \frac{T}{\theta}(\theta - \theta_e)$$

or approximately

$$(10) \quad -k_1 c_p (T - T_e)$$

It is also necessary to saturate the environmental air entrained into the updraft and a term of the form

$$(11) \quad -k_1 L (q - q_e)$$

is required, where L is the latent heat of vaporization. The loss of momentum per unit time per unit mass is

similar to that of the loss of heat and is assumed to be proportional to the difference between the velocity of the convective current and the environment, with the latter taken to be zero, with the result $-k_1 w$ where k_1 has the dimension second^{-1}. With this form for the friction term, eq (8) becomes

$$(12) \quad \frac{dw}{dz} = \frac{1}{(1+l)}\left[\frac{g}{w}\left(\frac{T_v - T_{ve}}{T_{ve}} - l\right) - (w - w_e)\frac{dN}{dz} - k_1\right]$$

The dimensions of k_1 imply that diffusive losses are independent of updraft velocity. Haltiner suggests a second approach to the problem of diffusion which makes the loss of heat and momentum dependent upon updraft velocity. The first approach, however, will be employed in this paper.

d. Thermodynamic equation.

In describing the thermodynamic process undergone by the rising air a number of factors must be included. The following must be taken into account in accordance with the First Law of Thermodynamics: the heat released by condensation, the heat required to saturate the entrained environmental air and to raise its temperature to that of the rising column, the heat lost to the environment through diffusion, the internal energy changes and the work done during expansion. The resulting equation embodying these factors is as follows:

$$(13)\quad -ML\,dq - L(q-q_e)\,dM - c_p(T-T_e)\,dM - Mk_1\left[c_p(T-T_e) + L(q-q_e)\right]dt = M\left[c_p\,dT - R_d T_v\,d(\ln p)\right]$$

The terms on the left, in the order of their occurrence, represent respectively, (a) the heat released in condensation; (b) the heat required to saturate the entrained air, (c) the heat required to bring the entrained air to the temperature of the rising column, (d) the heat lost through diffusion in the interval dt required for the air to rise a distance dz. The right side of the equation, represents the combined internal energy change and work by the gas in expanding from pressure p to pressure p + dp. By substitution from eq (4), eq (13) becomes, with appropriate manipulation,

$$(14)\quad -L\frac{dq}{dz} - \left[c_p(T-T_e) + L(q-q_e)\right]\frac{dN}{dz} - \frac{h_1}{w}\left[c_p(T-T_e) + L(q-q_e)\right] = c_p\frac{dT}{dz} + \frac{gT_v}{T_{ve}}$$

e. Moisture and liquid water.

It has been assumed that the rising air is saturated; the specific humidity of the rising air is then a function of temperature and pressure. As a good approximation we have the expression $q = .622\,e/p$ where e is the saturation vapor pressure. By differentiation this becomes

(15) $$\frac{1}{q}\frac{dq}{dz} = \frac{1}{e}\frac{de}{dz} - \frac{1}{p}\frac{dp}{dz}$$

The last term in eq (15) may be replaced with $g/R_d T_{ve}$ from eq (9), and from Clapeyron's equation, $\frac{de}{e} = \frac{L\,dT}{R_v T^2}$, where R_v is the gas constant for water vapor, an alternate expression for the second term is obtained, yielding

(16) $$\frac{1}{q}\frac{dq}{dz} = \frac{L}{R_v T^2}\frac{dT}{dz} + \frac{g}{R_d T_{ve}}$$

The liquid water content of the cloud is the amount of moisture condensed less the amount required to saturate the environmental air entrained and that exchanged through diffusion. Defining ℓ as the "specific humidity" of the liquid water we may write

(17) $$d(M\ell) = -M\,dq - (q-q_e)\,dM - Mk_1(q-q_e)\,dt$$

Expanding eq (17) yields

(18) $$\frac{d\ell}{dz} + \frac{dq}{dz} + (\ell+q-q)\frac{dN}{dz} + \frac{k_1}{w}(q-q_e) = 0$$

Equation (17), however, is not valid during detrainment and the change in liquid water content must be piecewise defined. For the detrainment phase we have

(19) $$d(M\ell) = -M\,dq + \ell\,dM - Mk_1(q-q_e)\,dt$$

which upon expansion leads to

(20) $$\frac{dl}{dz} = -\frac{dq}{dz} - \frac{k_1}{w}(q - q_e)$$

Two additional relationships between temperature and virtual temperature will be required. One of these is the well known relationship

(21) $$T_v = (1 + .61q)T$$

the second relationship is obtained from eq (21) by differentiating with respect to z; this yields

$$\frac{dT_v}{dz} = (1 + .61q)\frac{dT}{dz} + .61T\frac{dq}{dz}$$

Substituting from eq (16) this may be written

$$\frac{dT_v}{dz} = \left(1 + .61q + .61\frac{qL}{R_v T}\right)\frac{dT}{dz} + \frac{.61Tqg}{R_d T_{ve}}$$

By using some mean values this may be approximated by

(22) $$\frac{dT_v}{dz} = (1 + 12.5q)\frac{dT}{dz}$$

f. The environment.

The environmental parameters T_e, T_{ve}, q_e, and w_e appear in one or more of the equations derived above; it is therefore necessary to define the environment in some way. We have wide latitude in choosing the structure of the environment; it may, for instance, conform to an actual sounding. A simple model was used in this investigation. The lapse rate γ_e and the relative humidity were assumed constant, and w_e was assumed to

be zero. Under these assumptions T_e may be expressed as

$$(23) \quad T_e = T_{eo} - \gamma_e z$$

The environmental specific humidity may be determined as follows: From eq (16) the saturation specific humidity is given by

$$dq_{se} = \left(- \frac{L q_{se} \gamma_e}{R_v T_e^2} + \frac{g q_{se}}{R_d T_{ve}} \right) dz$$

and since the relative humidity is constant $q_e = r q_{se}$, we may write

$$(24) \quad dq_e = \left(- \frac{L q_e \gamma_e}{R_v T_e^2} + \frac{g q_e}{R_d T_{ve}} \right) dz$$

If the lapse rate γ_e, the initial temperature T_{eo}, and the relative humidity are chosen, then the values of T_e, T_{ve}, and q_e may be obtained at any level from equations (21), (23), and (24).

3. The complete model.

Having defined the environment in subsection 2 equations (5), (12), (14), (16), (21) and (18) or (20) constitute a system of six equations in 7 unknowns, w, T, T_v, q, l, N, and σ. We must now establish a relationship defining σ. The simplest solution, and the one which will be employed here, is to assume that σ is identically zero. This assumption is quite likely an oversimplification, but on the other hand presently available information on the behavior of the

cross-sectional area of an updraft is scanty and somewhat conflicting.

The system of equations (5), (12), (14), (16), (21), and (18) or (20) may be solved first for the cloud temperature lapse

$$(25)\quad \frac{dT}{dz} = \left\{ \frac{1}{2}\left[(q-q_e) + \frac{c_p}{L}(T-T_e)\right]\left[\frac{g}{w^2}\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right) - \frac{k_1}{w} - \frac{g}{R_d T_{ve}}\right] + \frac{k_1}{w}\left[(q-q_e) + \frac{c_p}{L}(T-T_e)\right] + \frac{g T_v}{L T_{ve}} + \frac{q g}{R_d T_{ve}} \right\} \left\{ -\frac{Lq}{R_v T^2} - \frac{c_p}{L} + \frac{(1+12.5q)}{2T_v}\left[(q-q_e) + \frac{c_p}{L}(T-T_e)\right] \right\}^{-1}$$

and then successively for the other parameters

$$(26)\quad \frac{dN}{dz} = \frac{1}{2}\left[\frac{g}{w^2}\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right) - \frac{k_1}{w} - \frac{g}{R_d T_{ve}} - \frac{1}{T_v}\frac{dT_v}{dz}\right]$$

$$(27)\quad \frac{dw}{dz} = \frac{1}{(1+\ell)}\left[-(w-w_e)\frac{dN}{dz} - k_1 + \frac{g}{w}\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right)\right]$$

$$(28)\quad \frac{dq}{dz} = \frac{Lq}{R_v T^2}\frac{dT}{dz} + \frac{q g}{R_d T_{ve}}$$

$$(29a)\quad \frac{d\ell}{dz} = -\frac{dq}{dz} - (\ell + q - q_e)\frac{dN}{dz} - \frac{k_1}{w}(q - q_e)$$

or

$$(29b)\quad \frac{d\ell}{dz} = -\frac{dq}{dz} - \frac{k_1}{w}(q - q_e)$$

Equations (21) through (24) are used as needed. Given initial values of T, N, w, ℓ, T_e, q, q_e, the foregoing system of equations may be integrated by numerical methods over the desired range of z.

Since adequate data concerning the vertical velocity of the entrained environmental air is lacking; eq (27) will be further simplified by assuming that $w_e = 0$. Moreover since typical values of ℓ are on the order of .005 we may place $1/(1+\ell) \doteq 1$. The error introduced by the latter approximation will be on the order of ½ of one percent.

If eq (27) is modified as indicated above, it becomes

$$(30) \quad \frac{dw}{dz} = - w \frac{dw}{dz} - k_1 + \frac{g}{w}\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right)$$

a form which differs from the corresponding equation in the Haltiner model only in the last term, which has the form

$$\frac{g}{w}\left(\frac{T_v - T_{ve}}{T_{ve}}\right)$$

Similarly in the Haltiner equations for dT/dz and dN/dz, the term $\frac{g}{w^2}\left(\frac{T_v - T_{ve}}{T_{ve}}\right)$ is replaced by $\frac{g}{w^2}\left(\frac{T_v - T_{ve}}{T_{ve}} - \ell\right)$ in the modified model. Equations (28) and (29a) are exactly the same as their counterparts in the Haltiner model. The only other difference in the two models is the appearance of the additional eq (29b).

4. Computations and Results.

The foregoing system of equations was integrated by numerical methods on an electronic digital computer, the National Cash Register 102A. With a given set of initial values, properly scaled inputs to the integration

routine were computed for the model given above. These values were then integrated over an interval of 10 meters yielding a new set of "initial" values for the level $z + 10$ meters which served as an input for the computation of a new set of inputs for the integration routine. Computation proceeded in this stepwise fashion until the top of the cloud was reached.

Computations were made for four sets of initial conditions. In each case the initial values corresponded to one of the sets of initial values used by Haltiner[4]. In Case (a) and Case (b) the same set of initial values were used, but in Case (a) only the effect of detrainment was considered. The effect of the drag of the liquid water was eliminated from Case (a) by an appropriate modification to the complete program. In Cases (b), (c), and (d), both the effects of detrainment and the drag of the liquid water are included.

The initial values used in the computation are as follows:

Case (a)

T_o = 20C, T_{eo} = 19°C, w_o = 1 meter/second
k_1 = .001/second, γ_e = .7 °C/100 meters
r = 80%

Case (b)

Same as Case (a) except that both detrainment and water drag are included.

Case (c)

Same as Case (b) except that $\gamma_e = .6^{\circ}$ C/100 meters

Case (d)

Same as Case (b) except that $r = 90\%$

The results of the computations are presented in figures 1 through 6 and table 1.

Figures 1 and 2 present the results of Cases (a) and (b), in which the parameters W, ℓ, M/M_0 and ΔT are plotted as functions of height. The parameter M/M_0 is the cumulative change in mass experienced by a unit of air due to entrainment. A value of two, for example, indicates that the mass has doubled. The parameter ΔT is defined as $T - T_e$.

Figures 3, 4, 5, and 6 present a comparison of the results obtained by Haltiner[4] for the set of values listed as Case (a) with the results of Cases (a) and (b). Each parameter is plotted separately as a function of height. The values computed by Haltiner and the values from Case (a) should be the same at and below the level where detrainment begins. The systematic difference in two values below this point is the result of minor differences in the computational procedures (primarily in the computation of the environmental parameters) followed in this investigation and those employed by Haltiner. This systematic difference is not significant

in itself, but it does illustrate how small differences at one level propagate upward and tend to accumulate.

The effects of detrainment and the drag of the liquid water are best illustrated in figures 4 and 5. It is apparent that the effect of detrainment alone is to accelerate the vertical current. The cloud height is greater in Case (a) than it is in either Case (b) or the corresponding case computed by Haltiner[4]. It is also apparent from the figures that in Case (a) the magnitude of the maximum values of w and M/M_0 and the elevation at which these maxima occur are greater than either the values obtained by Haltiner or the values of Case (b). These results, and the results of Cases (c) and (d) are summarized in table one.

5. Conclusions

The number of cases investigated was not exhaustive, but was sufficient to show that the modified model yields results that conform quite closely to the results of Haltiner[4]. In particular, the cloud heights were substantially the same but the vertical velocity and the amount of entrainment are reduced. These results are the natural consequence of the combination of detrainment and the drag of the liquid water. Detrainment on the one hand provides an acceleration, while the drag of the liquid water acts as a retarding force. These forces tend to balance one another, but the drag of the liquid

water somewhat overbalances the acceleration due to detrainment.

Haltiner [4] investigated an actual cloud for which observational data was available, and found that his results were in good agreement with the observed values. He did find, however, that his values of vertical velocity, although of the right order of magnitude, were somewhat in excess of the observed values. To the extent that the modified model reduces vertical velocity, the modifications appear to be a correction in the right direction.

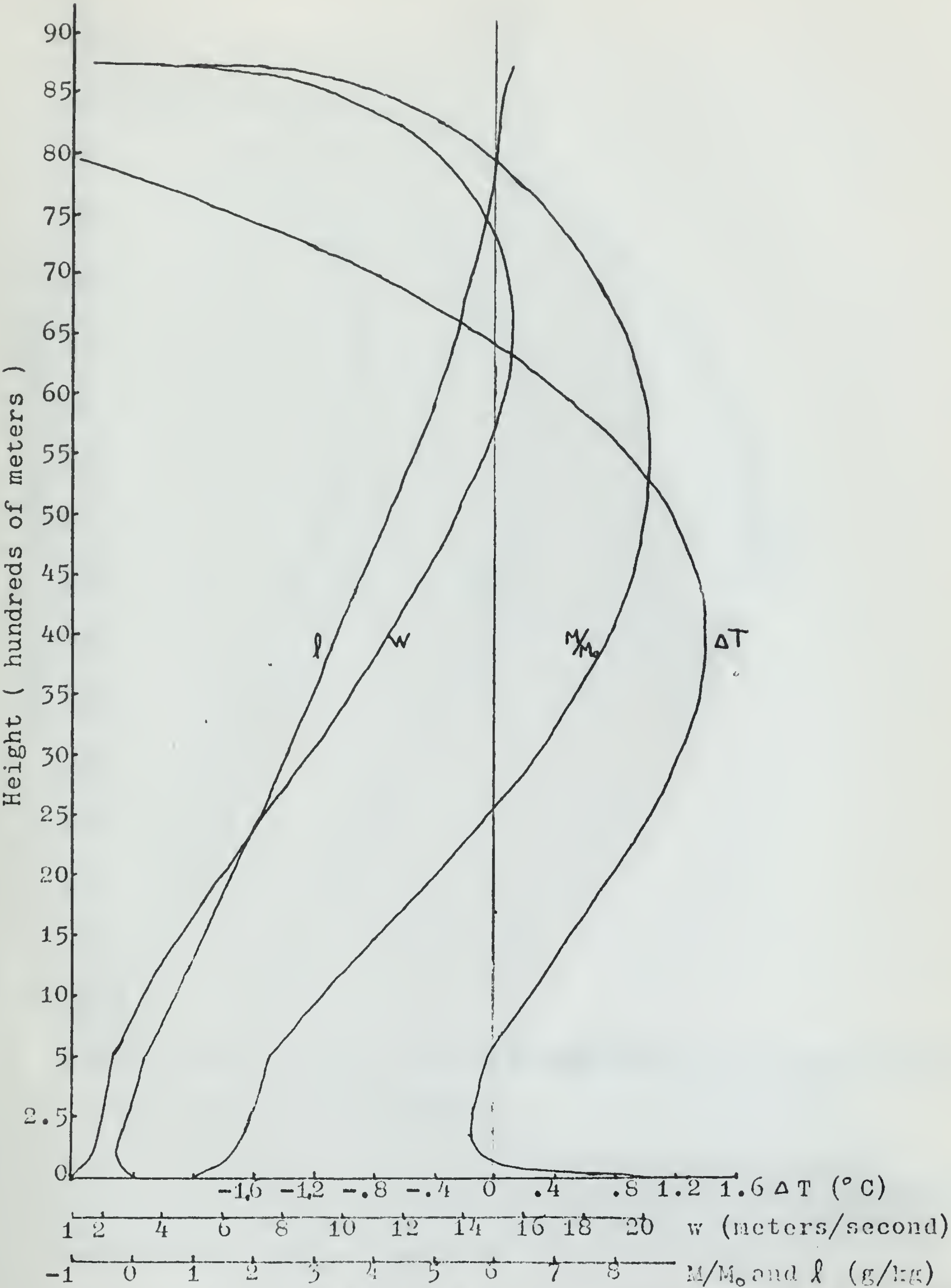


Figure 1. Vertical velocity w, fractional mass increase M/M_0, liquid water content ℓ and temperature excess over the environment ΔT as a function of height for Case (a).

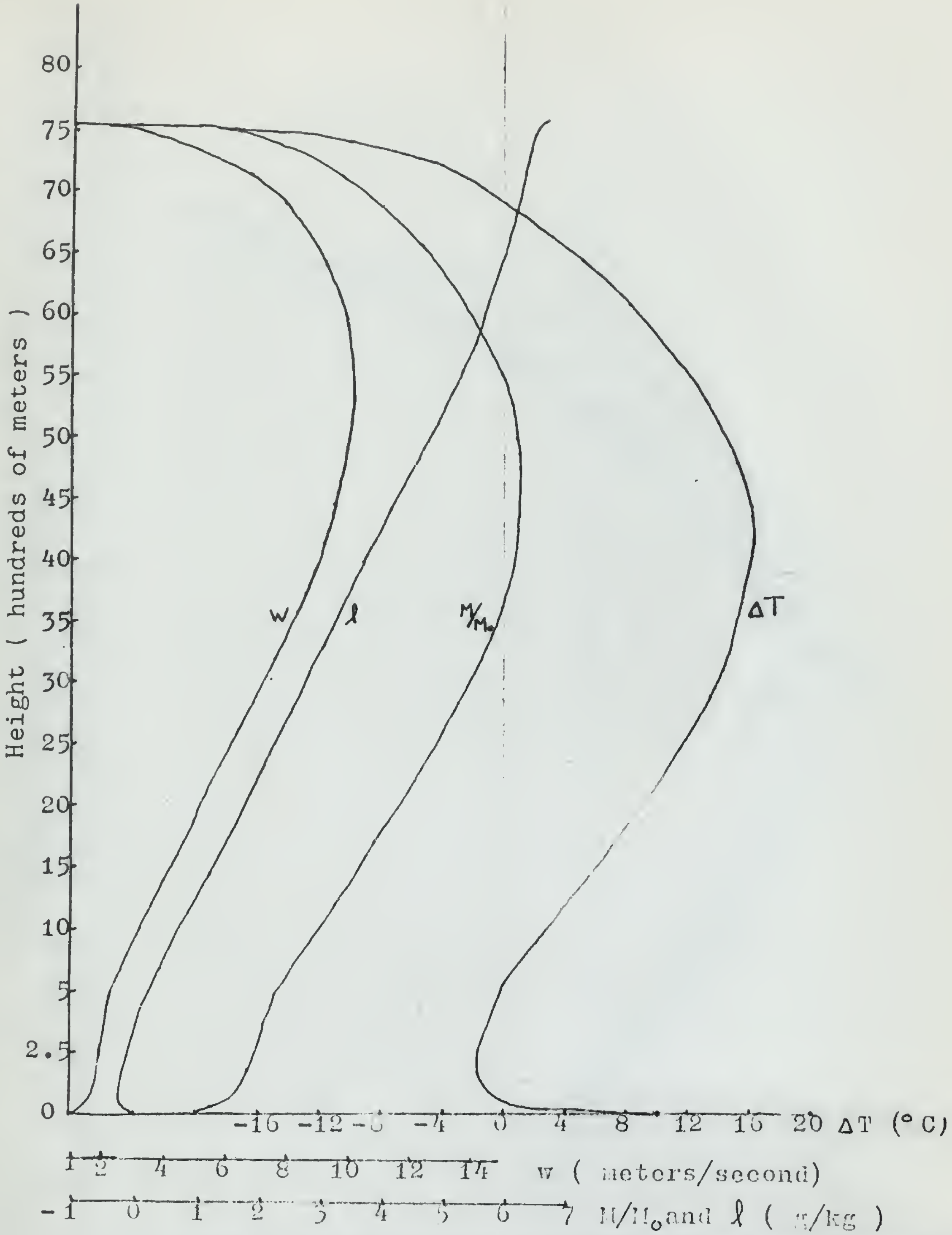


Figure 2. Vertical velocity w, fractional mass increase M/M_0, liquid water content ℓ and temperature excess over the environment ΔT as a function of height for case (b).

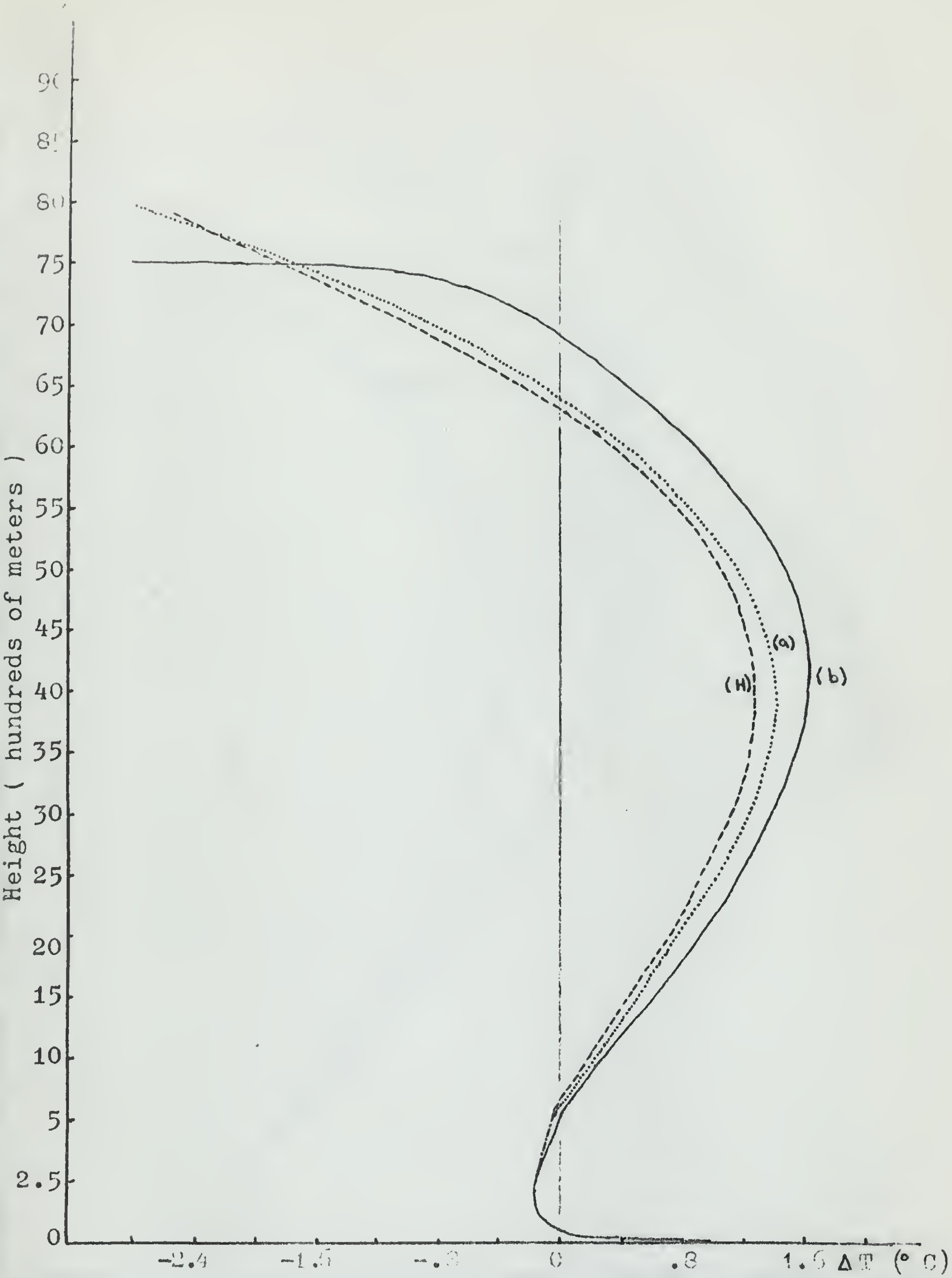


Figure 3. Temperature excess over environment ΔT as a function of height for Case (a), dotted, Case (b), solid, and after Haltiner, dashed.

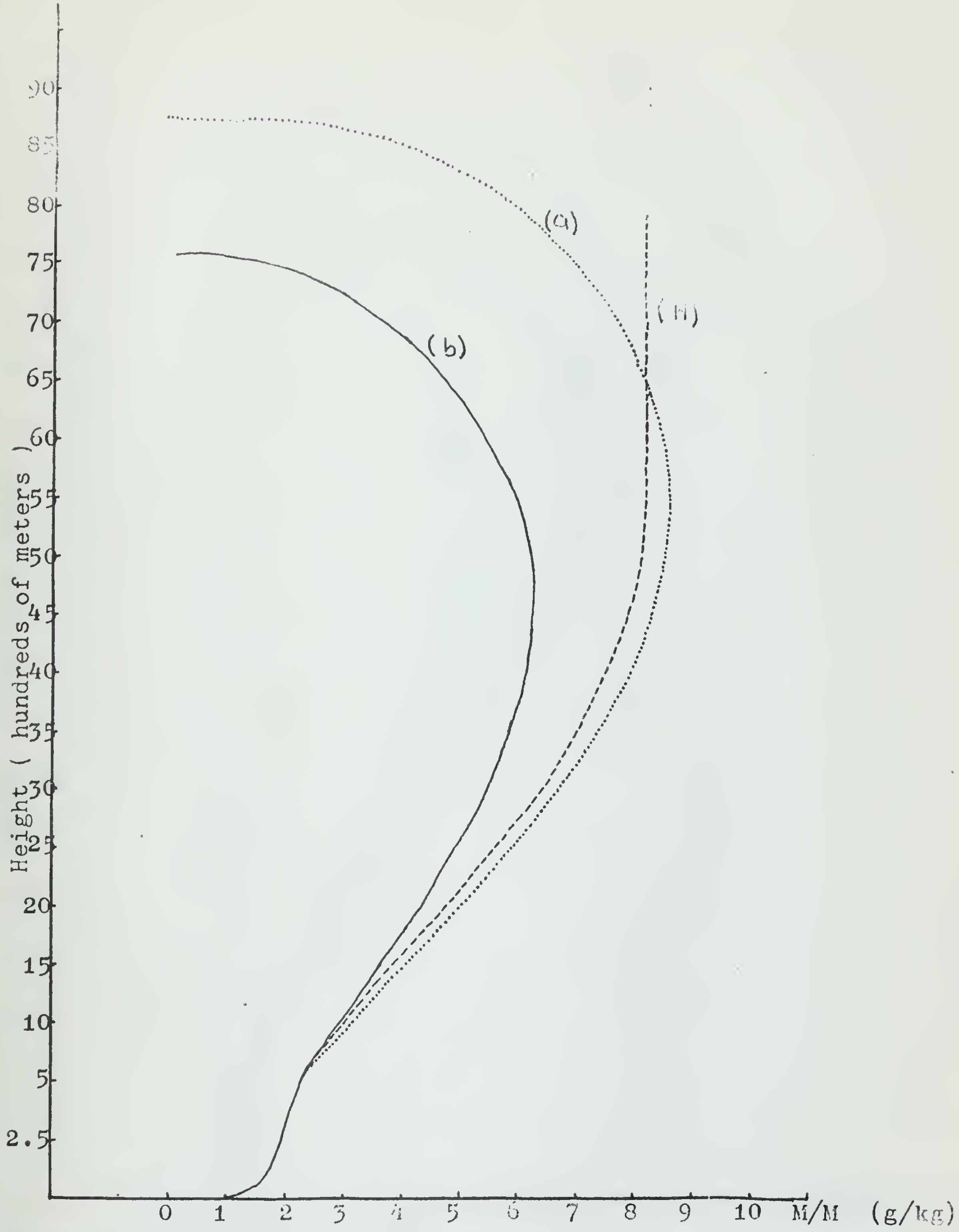
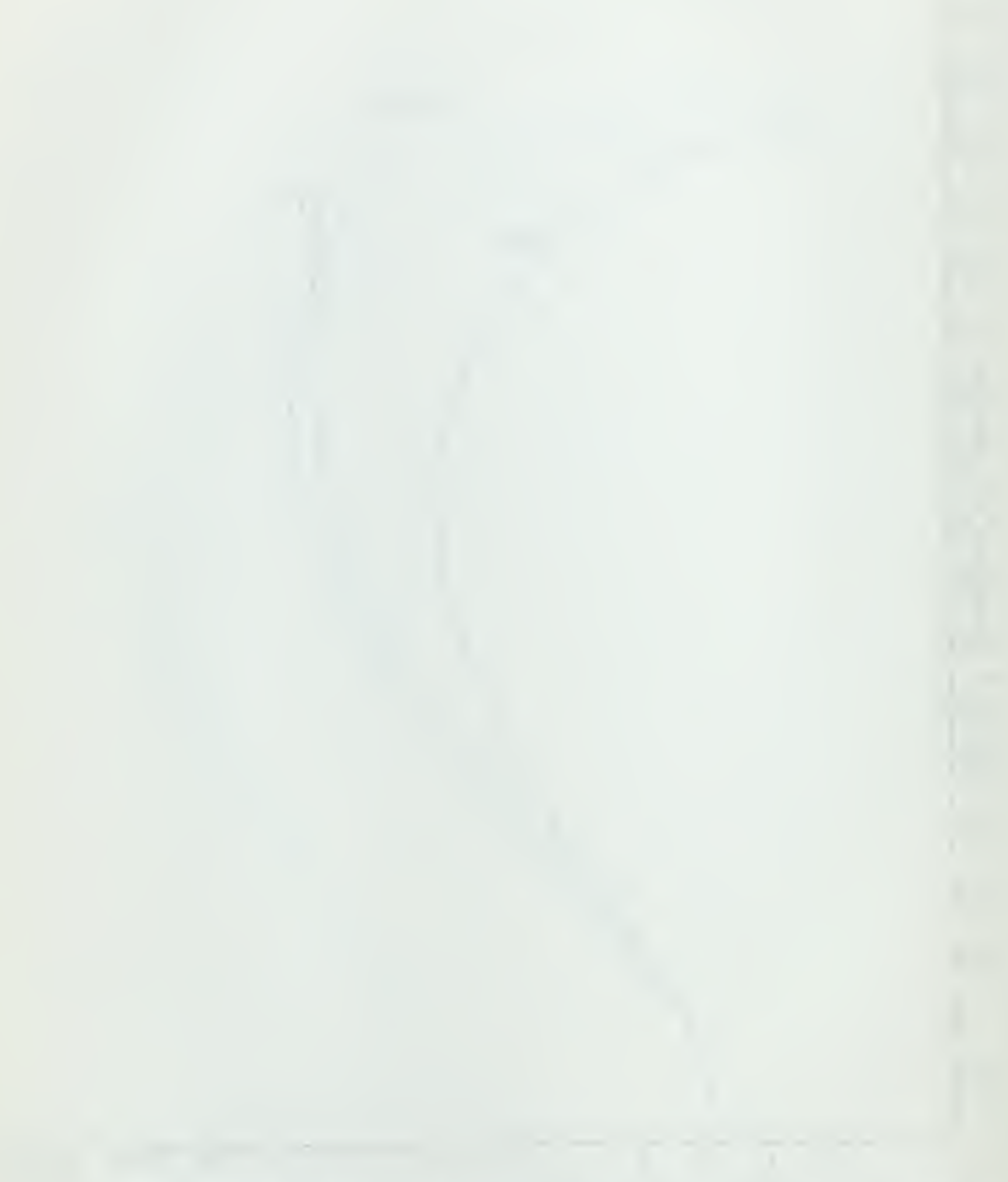


Figure 4. Fractional mass increase M/M_o as a function of height for Case (a), dotted, Case (b), solid, and after Haltiner, dashed.



[illegible]

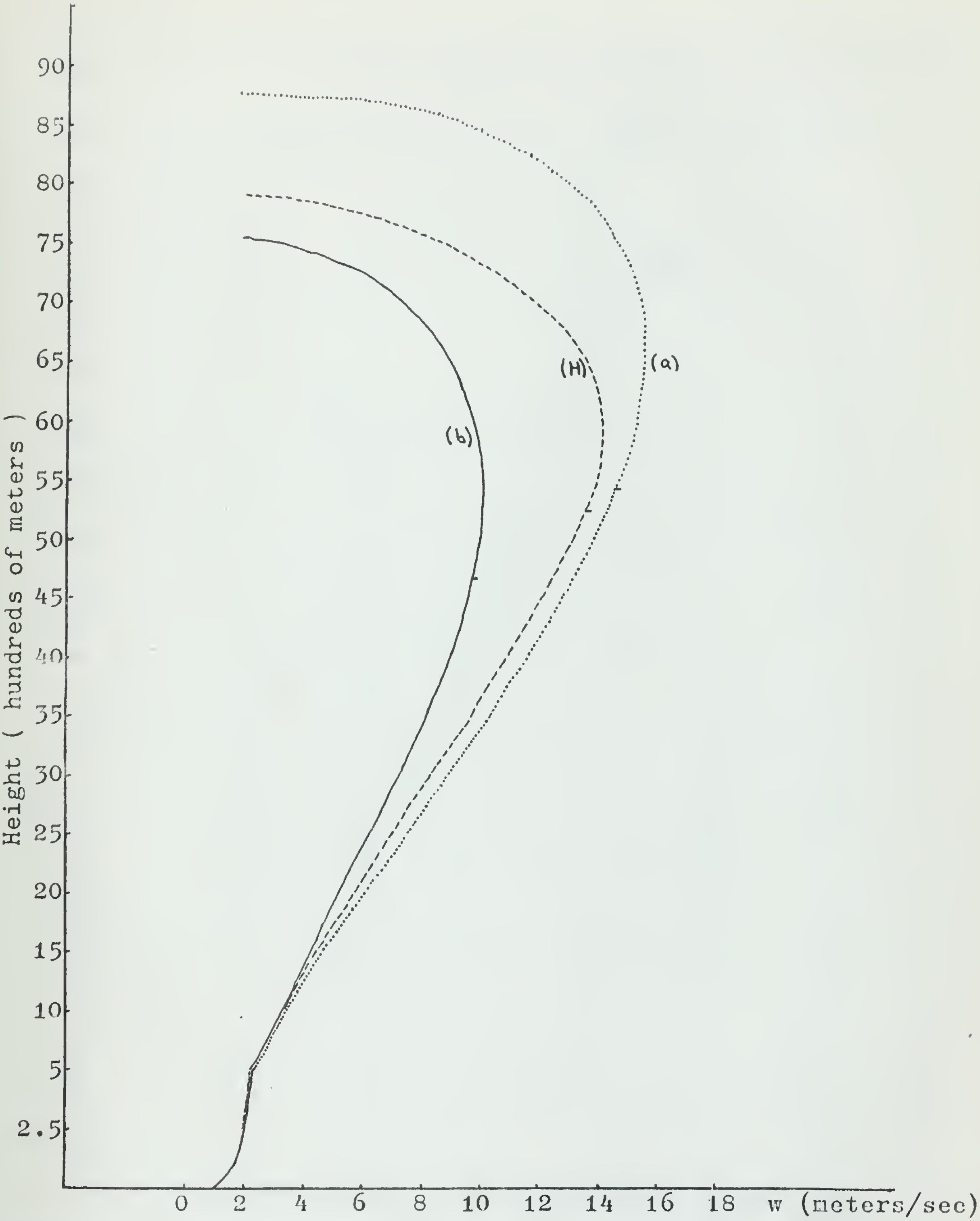


Figure 5. Vertical velocity, w, as a function of height for Case (a), Case (b) and after Haltiner. Tick marks on curves indicate level at which detrainment started.

[illegible]

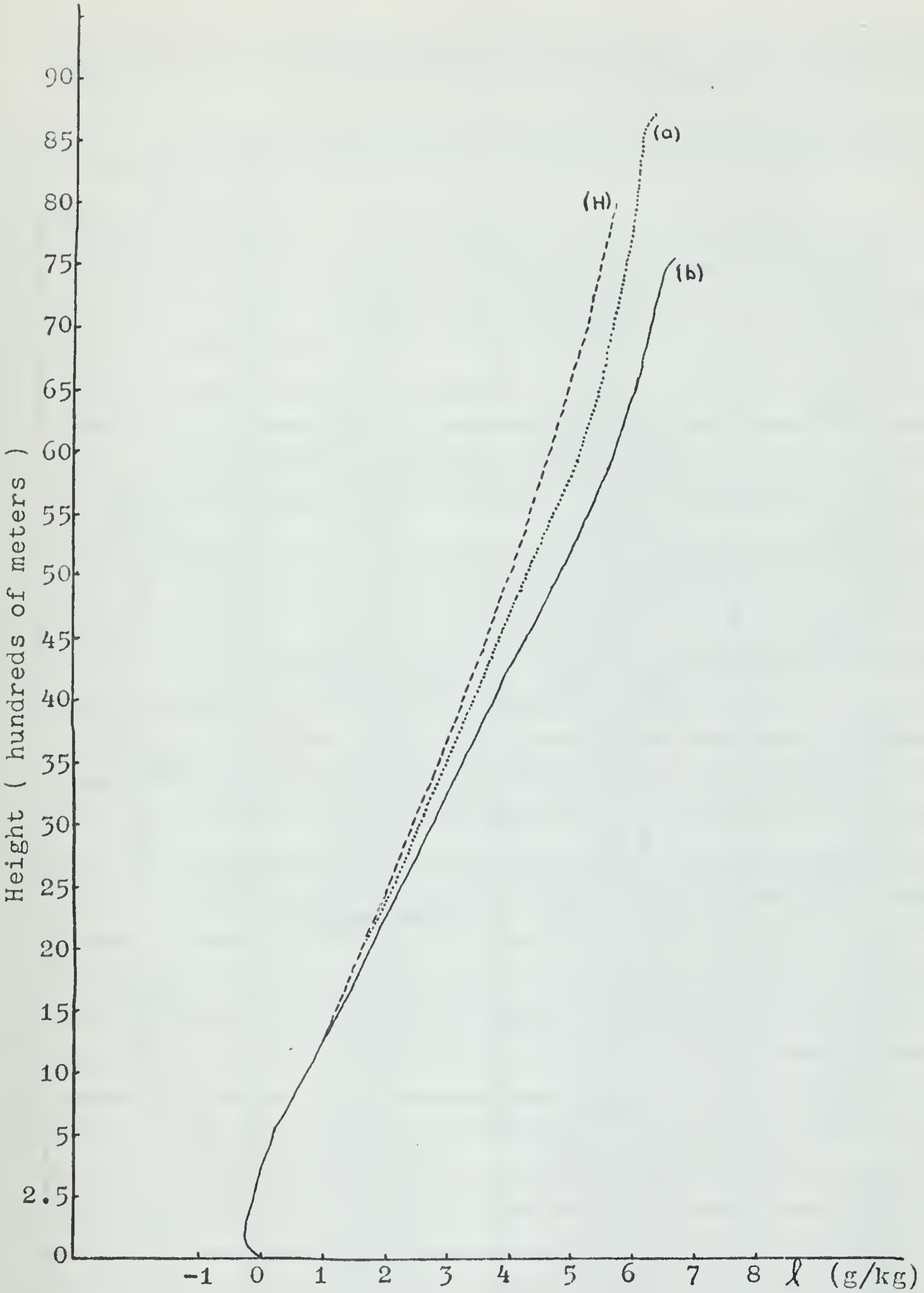


Figure 6. Liquid water content as a function of height for Case (a), dotted, Case (b), solid, and after Haltiner, dashed.

TABLE I

Comparison of maximum values of ΔT, M/M_0, ℓ, w and cloud height, h, with maximum values after Haltiner.

Case / Parameter	Haltiner	Case (a)	Difference
ΔT	1.26	1.39	.13
M/M_0	8.13	8.56	.43
w	14.12	15.52	1.40
ℓ	6.0	6.4	.4
h	7900	8750	850
	Haltiner	Case (b)	
ΔT	1.26	1.60	.34
M/M_0	8.13	6.22	1.81
w	14.12	10.13	3.99
ℓ	6.0	6.83	.83
h	7900	7550	350
	Case (a)	Case (b)	
ΔT	1.39	1.60	.21
M/M_0	8.56	6.22	2.34
w	15.52	10.13	5.39
ℓ	6.4	6.83	.43
h	8750	7550	1200
	Haltiner	Case (c)	
ΔT	-.01	+.97	.98
M/M_0	2.88	1.92	.96
w	4.07	2.12	1.95
ℓ	5.9	4.2	1.7
h	4750	4500	250
	Haltiner	Case (d)	
ΔT	2.47	2.76	.29
M/M_0	11.80	9.75	2.05
w	22.28	17.78	4.50
ℓ	7.1	7.7	.6
h	9500	9250	250

BIBLIOGRAPHY

1. Austin, J.M. and A. Fleisher, A Thermodynamic Analysis of Cumulus Convection, Journal of Meteorology, 5, pp 240-243, 1948.

2. Bunker, A.F., Diffusion, Entrainment, and Frictional Drag Associated with Non-saturated, Buoyant Air Parcels Rising through a Turbulent Air Mass, Journal of Meteorology, 10, pp.212-218, 1953.

3. Gill, S. A Process for the Step by Step Integration of Differential Equations in an Automatic Digital Computing Machine, The Cambridge Philosophical Society Meeting, 47, pp. 96-108, 1951.

4. Haltiner, G.J. On the Theory of Convective Currents, Tellus, 1, pp. 4-15, 1959.

5. Houghton, H.G. and H.E. Cramer, A Theory of Entrainment in Convective Currents, Journal of Meteorology, 8, pp. 95-102, 1951.

6. Malkus, J.S., Some Results of a Trade-cumulus Cloud Investigation, Journal of Meteorology, 11, pp. 220-237, 1954.

7. Priestley, C.H.B., A Working Theory of the Bentover Plume of Hot Gas, Quarterly Journal of the Royal Meteorological Society, 82, pp.165-177, 1956.

8. Stommel, H., Entrainment of Air into a cumulus Cloud, Journal of Meteorology, 4, pp.91-94, 1947.

thesC40
Further developments in the theory of co
3 2768 002 09739 6
DUDLEY KNOX LIBRARY